the diagrams. These are that the angle-of-attack disturbance is negligibly small in the phugoid mode, and the speed disturbance is negligible in the short-period mode. *They are essentially motions in two degrees of freedom* (see also Sec. 6.7).

Flight Paths in the Natural Modes

Additional insight into the modes is gained by studying the flight path of the airplane. This may be computed from Eqs. 4.14,6. For initially horizontal flight, $\theta_0 = 0$, and they become, for symmetric motion:

$$\frac{dx'}{dt} = u_0 + u = u_0(1 + \hat{u})$$

$$\frac{dy'}{dt} = 0$$

$$\frac{dz'}{dt} = -u_0\theta + w = u_0(-\theta + \alpha)$$

Using Eqs. 6.6,1 for flight in a natural mode, we get

$$\frac{1}{t^*}\frac{dx'}{d\hat{t}} = u_0(1 + \hat{u}_1 e^{n\hat{t}} \cos \omega\hat{t})$$

$$\frac{1}{t^*}\frac{dz'}{d\hat{t}} = u_0 e^{n\hat{t}}[\alpha_1 \cos(\omega\hat{t} + \varepsilon) - \theta_1 \cos(\omega\hat{t} + \delta)]$$

Integration yields

$$x' = u_0 t^*\left[\hat{t} + \frac{\hat{u}_1}{n^2 + \omega^2} e^{n\hat{t}}(n \cos \omega\hat{t} + \omega \sin \omega\hat{t})\right]$$

$$z' = u_0 t^* \frac{e^{n\hat{t}}}{n^2 + \omega^2}\{\alpha_1[n \cos(\omega\hat{t} + \varepsilon) + \omega \sin(\omega\hat{t} + \varepsilon) - \theta_1[n \cos(\omega\hat{t} + \delta) + \omega \sin(\omega\hat{t} + \delta)\}$$

By way of example, the amplitude of θ is assumed to be $\theta_1 = 0.2$ radian $= 11.46°$ for both modes. Using the results for the amplitude ratios and phase angles shown

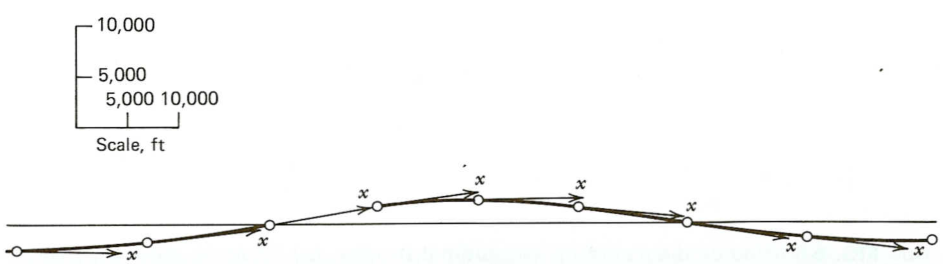

FIG. 6.7*a* Phugoid flight path (fixed reference frame).

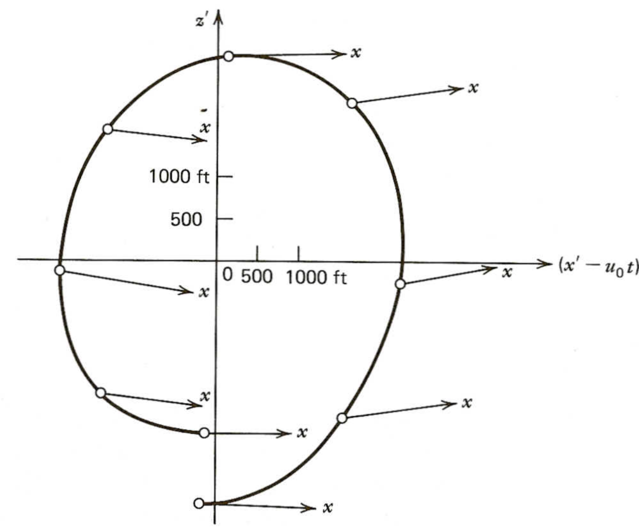

FIG. 6.7*b* Phugoid flight path (moving reference frame).

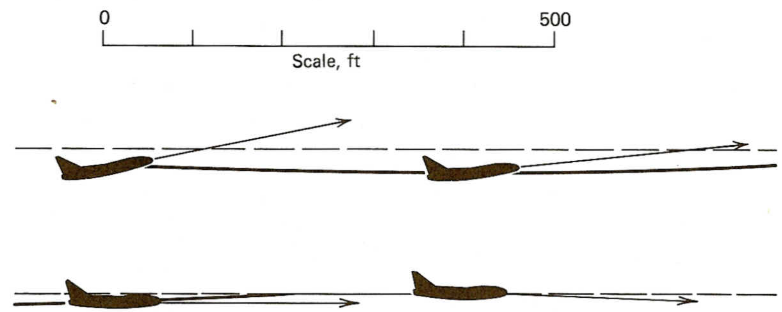

FIG. 6.7*c* Short-period flight path.

in Fig. 6.6, we have all the information required to compute the flight paths in the two modes. These computations have been carried out, and the results are displayed in Fig. 6.7. The physical characteristics of the modes are clearly evident in this figure. In the phugoid mode, the x axis of the airplane remains essentially tangent to the flight path, the principal feature of the motion being the slow rising and falling of the airplane, accompanied by change in speed. The speed is greatest when the height is least; there is a continual interchange between kinetic and potential energy. Figure 6.7*a* shows the path relative to fixed axes, and Fig. 6.7*b* shows the path as it would appear to an observer flying alongside at the steady speed u_0. At the scales of these figures, the airplane would be seen only as a point.

Figure 6.7*c* shows the initial part of the short-period motion. It is so rapidly damped out that the transient has virtually disappeared within 1,000 ft of flight. The deviation of the flight path from a straight line is small, the principal feature of the motion being the rapid rotation of the airplane in pitch.